Basic Multifrequency Tympanometry:
The Physical Background

by

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In tympanometry the mobility of the tympanic membrane is measured while the membrane is exposed to a (sinusoidal) tone of frequency f.

In the ear, the tympanic membrane is mechanically coupled with the middle ear ossicles to the oval window -the interface between middle and inner ear. It is this entire system (membrane, middle ear, oval window) that is forced into oscillation. The oscillation is detected by a microphone. (A more detailed description is given in "Tympanometry in just seconds" -http://www.grason-stadler.com/tymp.html.)

A linear theory used to evaluate the signal from the microphone is presented here. The response of a linear system when driven by a periodic oscillation can be expressed in terms of the resistance with which the system responds to the excitation (called "impedance") or in terms of the ease with which it is set into motion (called "admittance"). Both expressions of the response are presented here next to each other in a table.

An excellent summary of the results of the linear theory, its practical application and the reliability of multifrequency tympanometry in diagnosing middle ear diseases is: Robert H. Margolis, Lisa L. Hunter, Acoustic Immittance Measurements, Chapter 17 of Audiology: Diagnosis, by Ross J. Roeser, Michael Valente, Holly Hosford-Dunn (eds), Thieme 2000.

- **Chapter I** , "Forced Linear Oscillator", explains the definitions, assumptions and equations used to describe a linear oscillator forced into periodic movement by an external periodic force.
- **Chapter II**, "Acoustics", uses the theory presented in chapter I to derive a theory for multifrequency tympanometry.
- **Chapter III**, "Parameter determination from multifrequency tympanometry"
    - III.1,"Single resonance frequency system", explains how system parameters can be extracted from the multifrequency response of a single resonance system.
        - III.1.1, "Example", illustrates a system having just one resonance frequency.
    - III.2, "Coupled Systems" .
        - III.2.1, "Example: 2-Component system with subsystems arranged in parallel" presents results of a system with two single-component systems arranged in parallel
    - III.3 , "Fit of measured tympanometric data with linear model" analyzes an actual multifrequency tympanogram.

1. Forced Linear Oscillations

Mechanical impedance	**Mechanical admittance**
Assumption 1 (Hooke's law) Let m be a mass on a spring and F the force resulting from an elongation z of the spring. Then Hooke's law approximates the force F as being proportional to z $F_H = - D z$ (1) D is the Hook spring constant (compliance).	
Let t be the time variable.	
Assumption 2 (velocity proportional friction) When the mass moves, it is slowed down by friction R. Let the friction force be proportional to the speed of the movement $v = \frac{dz}{dt}$ $F_R = R \frac{dz}{dt}$ (2) where R is the friction coefficient.	

Definition 1 (periodic force) Let $F = F_0 e^{i\omega t} = F_0 [\cos(\omega t) + i \sin(\omega t)]$ (3) be a force wiggling at mass m with **frequency** $\omega = 2\pi$**f**. $i = \sqrt{-1}$ is the imaginary unit. F_0 is the amplitude of the force keeping the mass m in oscillatory motion.	
To wiggle at mass m, force F has to be the sum of • force $F_m = m \frac{d^2z}{dt^2}$, • the frictional force $F_R = R \frac{dz}{dt}$ and • the force of the spring $F_H = D z$.	
Theorem 1 (equation of motion) The resulting movement of the mass can be calculated from the force balance $F = F_m + F_R + F_H$, i.e. $m \frac{d^2z}{dt^2} + R \frac{dz}{dt} + D z = F_0 e^{i\omega t}$ (4)	

Assumption 3 (periodic movement) Let mass m oscillate with frequency ω and let this oscillation be out of phase (in comparison with the oscillation of force F) by α $z = z_0 e^{i(\omega t - \alpha)}$ (5)	**Mechanical admittance**
The velocity v of mass m is then $v = \frac{dz}{dt} = i \omega z_0 e^{i(\omega t - \alpha)}$ (6) Abbreviate $v_f = i \omega z_0 e^{i\omega t}$ (7)	
Theorem 2 (system response) The response of system (4) can be characterized by the ratio between force F and velocity v_f $\frac{F}{v_f} e^{i\alpha} = R + i (\omega m - \frac{D}{\omega})$ (8)	**Theorem 2a (alternative system response)** Alternatively, the system response can be characterized by the ratio between velocity v_f and force F $\frac{v_f}{F} e^{-i\alpha} = G + i B_{total}$ (8')
Proof: Plugging (5) into (4), and then dividing both sides of (4) with $\frac{dz}{dt} = i \omega z_0 e^{i(\omega t - \alpha)}$ yields $R + i\omega (m - \frac{D}{\omega^2}) = \frac{F_0 e^{i\omega t}}{i\omega z_0 e^{i(\omega t - \alpha)}}$ (9) Substituting (3) and (7) in (9) give us (8).	*Proof:* The reciprocal of (9) is $\frac{R}{R^2 + (m\omega - \frac{D}{\omega})^2} + i \frac{-m\omega + \frac{D}{\omega}}{R^2 + (m\omega - \frac{D}{\omega})^2} = \frac{i \omega z_0 e^{i(\omega t-\alpha)}}{F_0 e^{i\omega t}}$ Substituting (3) and (7), this can be written as $G + i B_{total} = \frac{v_f}{F} e^{-i\alpha}$

Definition 2 (impedance) $\frac{F}{v_f} e^{i\alpha} = Z_m$ (10) This ratio (10) will be called **mechanical impedance** Z_m. The real and imaginary parts of the sum have the following names:	**Definition 2a (admittance)** $\frac{v_f}{F} e^{-i\alpha} = Y$ (10') In (10') the following abbreviations are used:
	admittance $Y = \frac{1}{R^2 + (m\omega - \frac{D}{\omega})^2} \{R + i\,(-m\omega + \frac{D}{\omega})\}$
resistance R	**conductance** $G = \frac{R}{R^2 + (m\omega - \frac{D}{\omega})^2}$
reactance $X_{total} = X_m + X_c$	**susceptance** $B_{total} = B_m + B_c$, with
mass reactance $X_m = m\,\omega$	**mass susceptance** $B_m = -\frac{m\omega}{R^2 + (m\omega - \frac{D}{\omega})^2}$
compliant reactance $X_c = -\frac{D}{\omega}$	**compliant susceptance** $B_c = \frac{\frac{D}{\omega}}{R^2 + (m\omega - \frac{D}{\omega})^2}$

II. Acoustics

Acoustic impedance Let δV be a fast (adiabatic, i.e. heat non-dissipating) change of a volume V of air and δP the corresponding pressure change.	**Acoustic admittance**
Definition 3 (compressibility κ) The adiabatic compressibility of air is defined as $\frac{\delta V}{V} = -\kappa\,\delta P$ (11)	
Theorem 3 The compressibility can be expressed in terms of the density ρ of the air and the speed of sound c in air: $\kappa = \frac{1}{\rho\, c^2}$ (12) Proof can be found in textbooks of physics.	
Let volume V be approximated by a cylinder with base A (and a height h).	
Definition 4 (cross section A of air volume) Then volume change δV can be expressed as change z of the cylinder height $\delta V = A\,z$. (13)	
The corresponding pressure change δP can be written in terms of the force F on A $\delta P = \frac{F}{A}$ (14)	

Definition 5 (Hooke's constant D for air, acoustic stiffness K_a) Combining (11) - (14) the force F resulting from the volume change δV can be written similarly as Hooke's law $F_H = - D z$ (15) with the abbreviation $D = \frac{\rho c^2}{V} A^2 = K_a A^2$ units of D: $\frac{g}{s^2}$ (16) where (see (11), (12)) $K_a = \frac{\delta p}{\delta V}$	**Acoustic admittance**
Assumption 4 (friction R) Let the volume V of air dissipate energy similarly as the mass m on a spring in (2): $F_R = R \frac{dz}{dt}$	
Assumption (rigid body of oscillating masses) The periodic oscillation of the air in the ear canal wiggles at the tympanic membrane, the middle ear ossicles etc. This has been ignored in the system dealt with until now. Let us assume that all those masses comprise a rigid entity m_{eff} that oscillates as a whole and in phase with the air in the ear canal. In other words, the masses of which m_{eff} is composed do not oscillate separately and out of phase with the air. **Definition 7 (oscillating mass m)** The total oscillating mass m is therefore the mass ρV of the air plus the effective mass: $m = \rho V + m_{eff}$ Thus, the force to overcome the inertia of m is $F_m = m \frac{d^2 z}{dt^2}$ (17)	
Theorem 4 (equation of motion) As in the case of the mechanical oscillator, the resulting movement of the air particles in volume V can be calculated from the force balance $F = F_m + F_R + F_H$, where $F = A p = A p_0 e^{i\omega t}$ $m \frac{d^2 z}{dt^2} + R \frac{dz}{dt} + D z = A p_0 e^{i\omega t}$ (18) $m \frac{d^2 z}{dt^2} + R \frac{dz}{dt} + A^2 K_a z = A p_0 e^{i\omega t}$ (18a)	
For a periodic pressure being applied by a loudspeaker to the ear canal air and mass m_{eff} (assumption 3), the system's response is analogous to (8) (note that again F = A p): $\frac{A p}{v_f} e^{i\alpha} = R + i (m \omega - \frac{D}{\omega})$. (19)	

Definition 6 (volume velocity U) Volume velocity U is defined as the volume that flows through the air canal cross section per unit time: $A v_f = i \omega A z_0 e^{i\omega t} = i U$	**Acoustic admittance**

It is customary to replace v_f in (19) with iU/A. Deviding both sides of (19) by A^2 we get the following expression chaaracterizing the system response

$$\frac{p}{i\,U}\, e^{i\alpha} = \frac{R}{A^2} + i\,(\omega \frac{m}{A^2} - \frac{D}{\omega A^2}) \qquad (20)$$

Definitions 7 (R_a, acoustic inertance M)

(1) To simplify the form of the equations, we will introduce the abbreviation

$$R_a = \frac{R}{A^2}.$$

(2) Likewise, Kinsler and Frey (1962, p. 190, Eq. 8.14) introduced the definition of acoustic inertance

$$M = \frac{m}{A^2}.$$

Using (16), the last term on the right hand side can be simplified:

$$\frac{D}{\omega A^2} = \frac{\rho\, c^2}{\omega V} = \frac{K_a}{\omega} \qquad (21)$$

Theorem 5 (system response)
The final expression for the system response is (20). In analogy with (10) the ratio (22) is called acoustic impedance Z_a

$$Z_a = \frac{p}{i\,U}\, e^{i\alpha} \qquad (22)$$

Theorem 5a (alternative system response)
Alternatively, the system response can be characterized by the inverse of ratio (22)

$$Y_a = G_a + i\,B_a = i\,\frac{U}{p_0}\, e^{-i\alpha} \qquad (22')$$

Definition 8 (acoustic impedance Z_a, eqs. (23))
The impedance Z_a given in (22) has a real and an imaginary part (see (20)).

$Z_a = R_a + i\,(X_{ma} + X_{ca})$, ... unit: $\frac{g}{cm^4\, s}$ = ohm

With definitions 7.1 (acoustic resistance Ra) and 7.2 (acoustic inertance M) and definition 5 (acoustic stiffness Ka) the acoustic impedance can be written in analogy with definition 2, and the following names are given:

R_a ....**resistance**

$X_a = X_{ma} + X_{ca}$ ....**reactance**

$X_{ma} = \omega \frac{m}{A^2} = \omega M$ ....**mass reactance**

$X_{ca} = - \frac{\rho\, c^2}{\omega V} = - \frac{K_a}{\omega}$ ....**compliant reactance**

Definition 8' (acoustic admittance Y_a, eqs. (23'))

$Y_a = G_a + i\,B_a = G_a + i\,(B_{ma} + B_{ca})$, ... unit: $\frac{cm^4\, s}{g} = \frac{1}{\Omega}$

$10^{-3}\, \frac{1}{\Omega}$ = 1 mmho

$G_a = \frac{R_a}{R_a^2 + X_a^2}$ ....**conductance**

$B_a = - \frac{X_a}{R_a^2 + X_a^2}$ .... **susceptance**

$B_{ma} = - \frac{X_{ma}}{R_a^2 + X_a^2}$ .... **mass susceptance**

$B_{ca} = - \frac{X_{ca}}{R_a^2 + X_a^2}$ .... **compliant susceptance or stiffness susceptance**

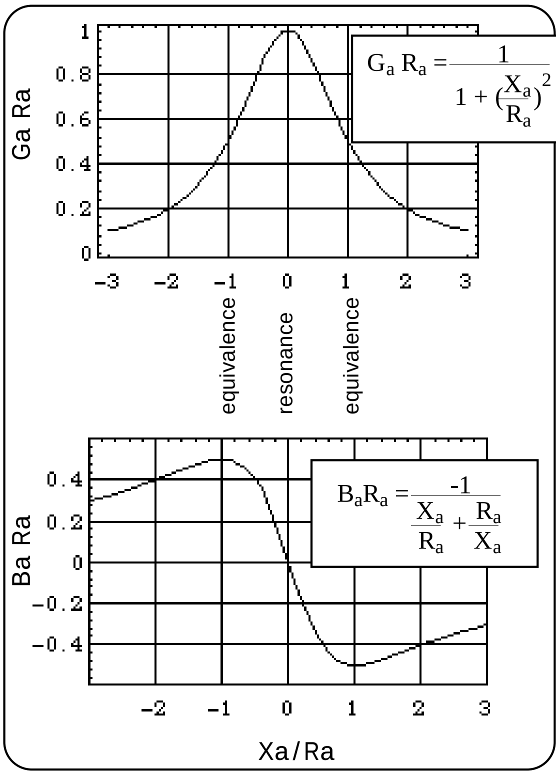


Fig. 1: G_aR_a and B_aRa as a funtion of X_a/R_a. At $|X_a|/R_a = 1$ $G_a R_a$ and $B_a R_a$ have the same size. At resonance $G_aR_a = 1$ and $B_aR_a = 0$.

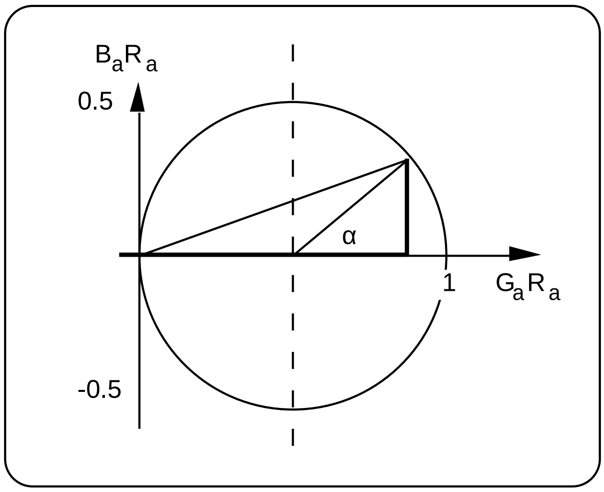


Fig. 2: Oscillation plotted in $\{G_aR_a, B_aRa\}$ plane lies on a circle with radius 1/2, because $G_aR_a{}^2 + B_aR_a{}^2 = (1/2)^2$ for all X_a/R_a. The angle α will be used to calculate R_a from G_a and B_a.

Fit of R_a to multifrequency tympanogram $G_a(f)$ and $B_a(f)$

Definition of α (see Fig. 2)

$$G_aR_a - \frac{1}{2} = \frac{1}{2}\cos\alpha \qquad (24)$$

$$B_aR_a = \frac{1}{2}\sin\alpha \qquad (25)$$

From (23), (24) follows

$$\frac{G_a}{B_a} = \frac{1+\cos\alpha}{\sin\alpha} \qquad (26)$$

Proof:

$$\frac{(24)}{(25)} = \frac{2\,G_aR_a}{2\,B_aR_a} = \frac{1+\cos\alpha}{\sin\alpha}$$

Multifrequency tympanogramm gives $G_a(f)$ and $B_a(f)$. Thus (26) is a function of the immission frequency f. (26) can be solved for α as a function of f.

With (25) R_a can be fitted to the tympanogram

$$R_a(f) = \frac{\sin\alpha(f)}{2\,B_a(f)} \qquad (27)$$

Definition 9 (resonance frequency ω_r)

Let the frequency at which the reactance X_a and susceptance B_a vanish be called resonance frequency ω_r of the system:

$\omega_r \frac{m}{A^2} = \frac{\rho\, c^2}{\omega_r V}$. Solving for ω_r

$$\omega_r = 2\pi f_r = A\, c\sqrt{\frac{\rho}{V\, m}} \qquad (28)$$

At resonance ω_r conductance and resistance are simple reciprocals of each other:

$$G_a = \frac{1}{R_a}$$

Data:

$$\rho\, c^2 = 1.42\ 10^6\ \frac{g}{cm\ s^2} \qquad (d1)$$

At f = 226 Hz

$$\omega = 2\pi\, f = 1.42\ 10^3\ s^{-1} \qquad (d2)$$

Plugging (d1) and (d2) into the definition of X_{ca} above

$$X_{ca} = -\frac{\rho\, c^2}{\omega\, V} = -\frac{10^3 g}{cm\ s}\frac{1}{V} \qquad (d3)$$

A volume V = 1 cm^3 of air has a compliant reactance

$$X_{ca} = -\frac{10^3 g}{cm^4\ s} = 10^3 ohm$$

At **high** positive or negative ear canal pressures the tympanic membrane is almost fixed and the middle ear is nearly motionless ($m_{eff} \approx 0$, $R_a \approx 0$) the admittance $Y_a \approx B_a \approx B_{ca}$ (the latter because $X_{ma} << X_{ca}$) with

$$B_{ca} = \frac{\omega\, V}{\rho\, c^2}$$

Since B_{ca} can be determined experimentally, the ear canal volume V can be calculated from this equation. At f = 226 Hz

$$B_{ca} = 10^{-3}\ \frac{cm\ s}{g}\ V.$$

A volume V = 1 cm^3 of air has a compliant susceptance

$$B_{ca} = 10^{-3}\ \frac{cm^4\ s}{g} = 1\ mmho$$

III. Parameter determination from multifrequency tympanometry

III.1 Single resonance frequency system

Fit of V and m/A^2 to X_a/R_a

Use definitions (23) of X_{ma} and X_{ca}:

$$X_{ma} = \omega \frac{m}{A^2} \qquad (23.1)$$

$$X_{ca} = -\frac{\rho c^2}{\omega V} = -\frac{K_a}{\omega} \qquad (23.2)$$

- Plot $\log|X_a|$ as a funtion of logf as shown below.
- Intersections of asymptotic lines with y-axis at logf = 0 give
  - V and
  - m/A^2.

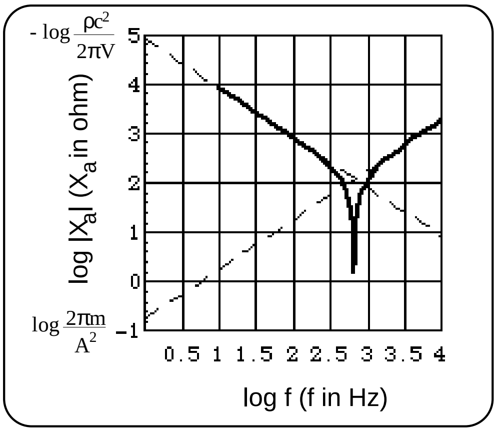


Fig. 3: Extrapolation of $X_{ca}(f)$ and $X_m(f)$ yields V and m/A.

Another possibility: Ear canal cross section A together with oscillating mass m can be fitted to the resonance frequency f_r.

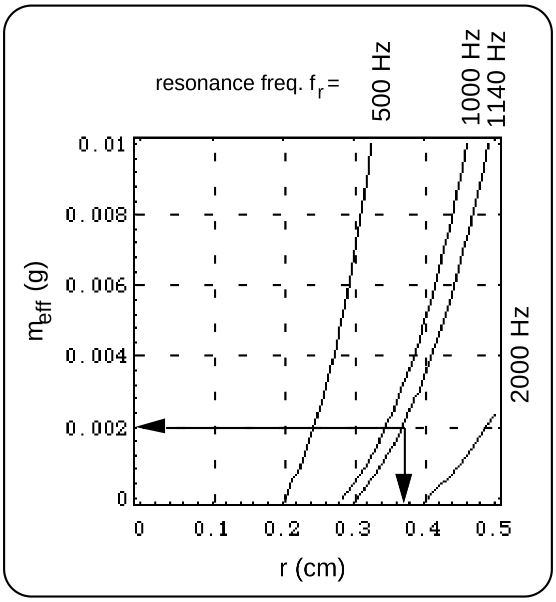


Fig. 4: Plot of contours of constant resonance frequency f_r as a funtion of the ear canal radius r and the oscillating effective mass m_{eff}.
Example marked by arrows: for r = 0.37 cm and m_{eff} = 0.002 g the resonance frequency is f_r = 1140 Hz.

As the contour plot Fig. 4 shows, a possible choice for
f_r = 1140 Hz
is
$A = r^2 \pi = (0.37\ \text{cm})^2 \pi$
$\rho = 0.00129\ \text{g/cm}^3$
$V = 1.36\ \text{cm}^3$
m = $\rho V + m_{eff}$ = (0.0018 + 0.002) g = **0.0038 g**.

III.1.1 Example

Choice of dependence of V on ear canal pressure p**:**

$$V(p) = \frac{V_0}{2}\left(1 + e^{-\frac{|p|}{TW}}\right), \qquad (29)$$

$$m(p) = \rho\, V(p) + m_{eff}\, e^{-\frac{|p|}{TW}}. \qquad (30)$$

Data used in Example:

$V_0 = 1.36\ \text{cm}^3$, TW = 40 daPa = 400 Pa (1daPa = 10 Pa)
$m_{eff} = 0.002$ g, $R_a = 1000$ ohm, r = 0.37 cm, $\rho = 0.00129\ \text{g/cm}^3$. (31)

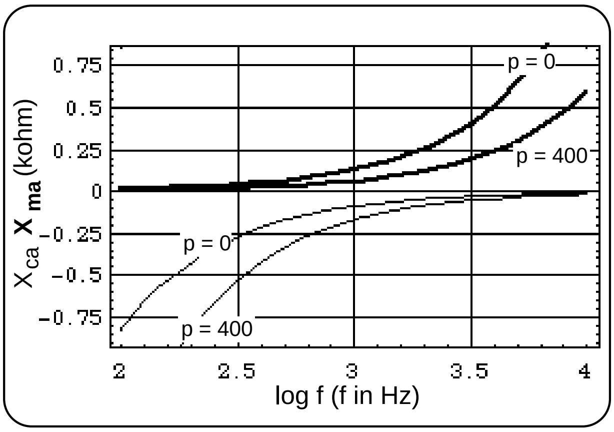


Fig. 5: Plot of the two components of the reactance as functions of immission frequency f. Heavy curves represent mass reactances, light curves compliant reactances. Curve parameter is the ear canal pressure. Curves are plotted for p = 0 and p = 400 daPa. (for implementation of p see (29) and (30)).

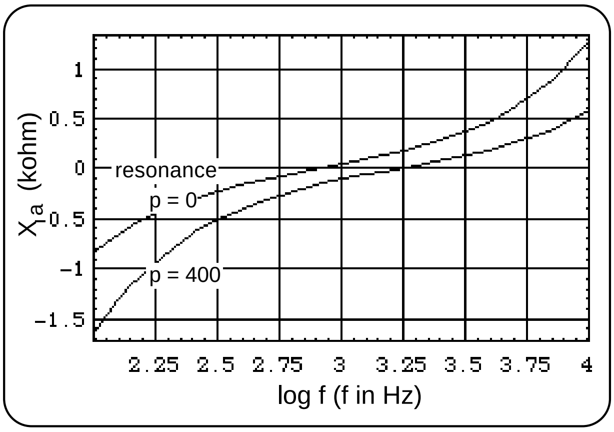


Fig. 6: Plot of total reactance as a function of immission frequency f for fixed ear canal pressures p = 0 and p = 400 daPa. At resonance X_a is 0.

Definitions:

- resonance: $X_a = 0$.
- equivalence: $|X_a|/R_a = 1$.

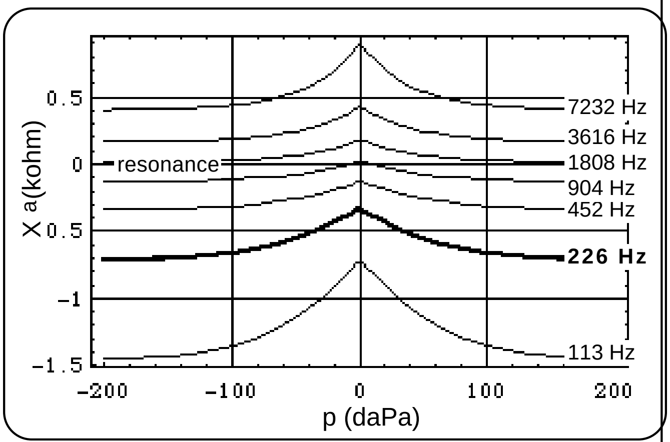
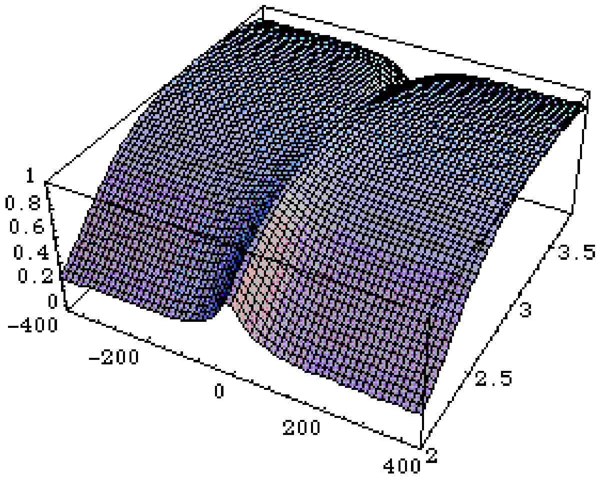


Fig. 7: Plot of total reactance as a function of ear canal pressure p. Curve parameter is the immission frequency f. Curves are plotted for f =113 Hz and the following 6 octaves above 113 Hz.

Conductance G_aR_a



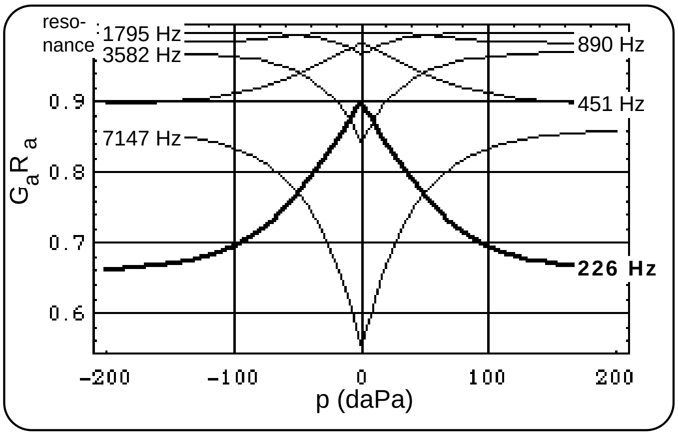
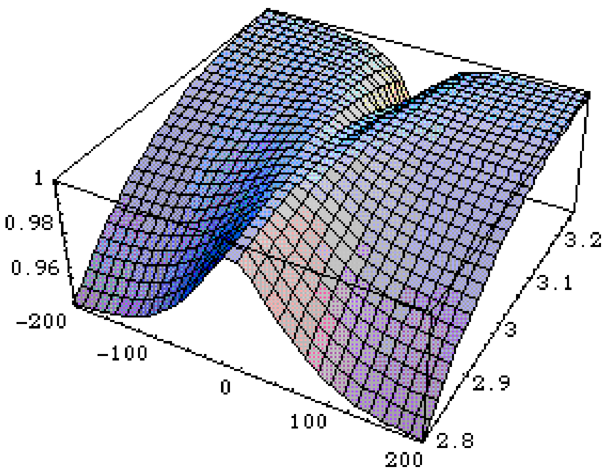


Fig. 8: G_aR_a as a function of both ear canal pressure p (daPa) and immission frequency f (log f is used, with f in Hz).
Left: 3D-plot, p is plotted along the x-axis (range: -400 daPa ≤p ≤ 400 daPa), log f is plotted along the y-axis (range: 2 ≤ log f ≤ 3.7).
Right: 2D-plot $G_aR_a(p)$ with f as parameter, i.e for f fixed at 226 Hz and the 5 following octaves above 226 Hz.

- near resonance



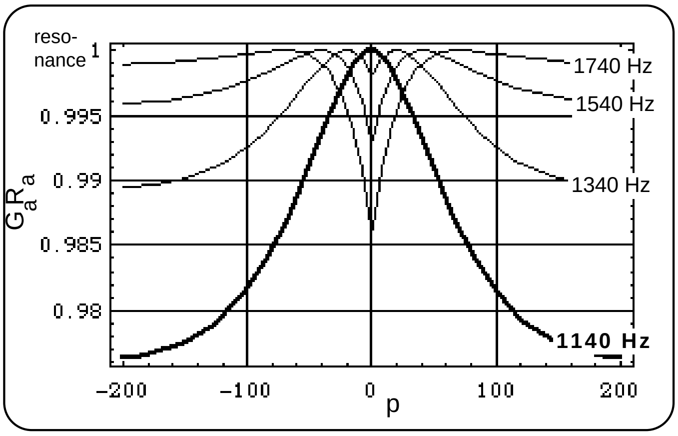


Fig. 9: Detail of Fig. 9 near resonance at zero ear canal pressure p = 0.

Susceptance B_aR_a

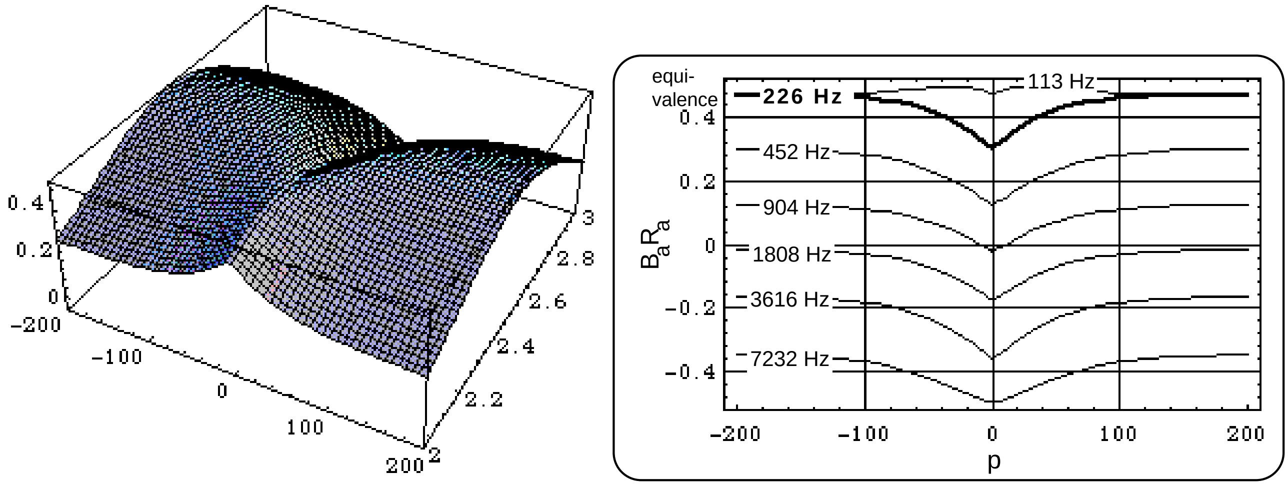


Fig. 10: B_aR_a as a function of both ear canal pressure p (daPa) and immission frequency f (log f is used, with f in Hz).
Left: 3D-plot, p is plotted along the x-axis (range: -400 daPa ≤p ≤ 400 daPa), log f is plotted along the y-axis (range: 2 ≤ log f ≤ 3.7).
Right: 2D-plot $B_aR_a(p)$ with f as parameter.

- near equivalence frequency

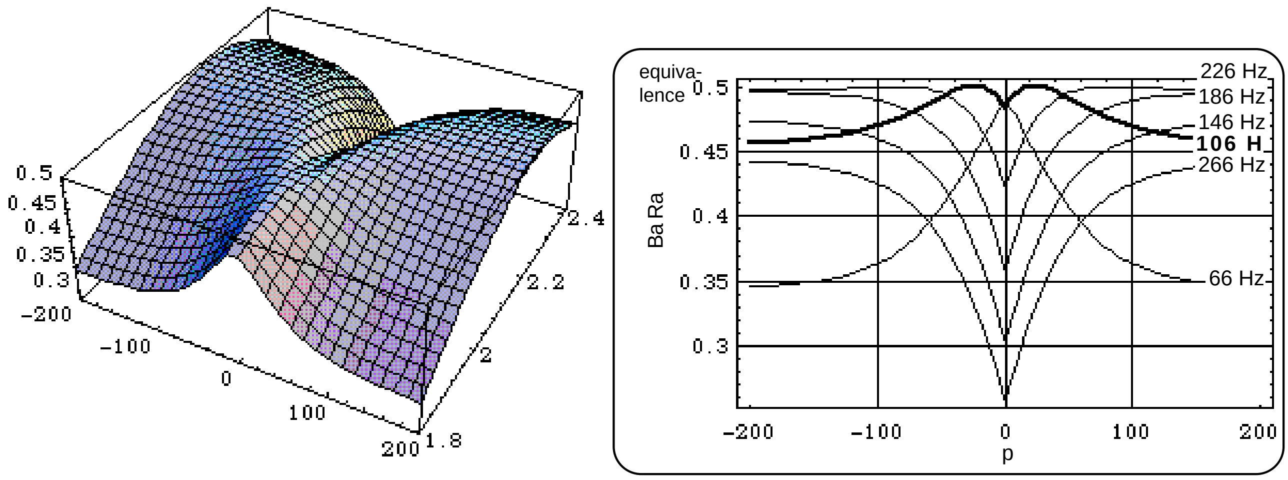


Fig. 11: Detail of Fig. 10 near equivalence frequency at zero ear canal pressure p = 0.

Admitttance Y_aR_a

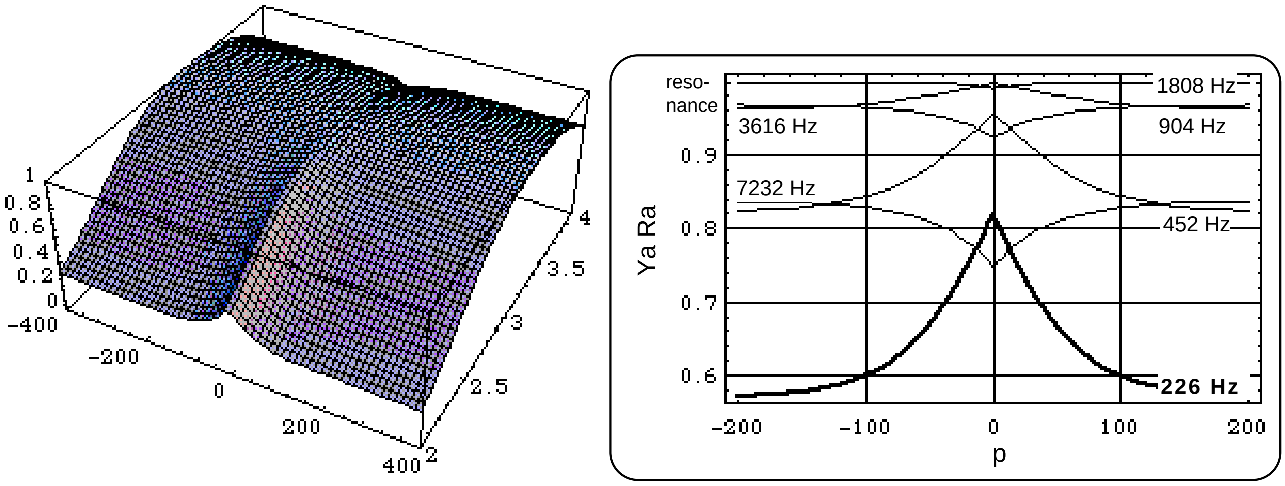


Fig. 12: Y_aR_a as a function of both ear canal pressure p (daPa) and immission frequency f (log f is used, with f in Hz).
Left: 3D-plot, p is plotted along the x-axis (range: -400 daPa ≤p ≤ 400 daPa), log f is plotted along the y-axis (range: 2 ≤ log f ≤ 3.7).
Right: 2D-plot $Y_aR_a(p)$ with f as parameter.

- near resonance

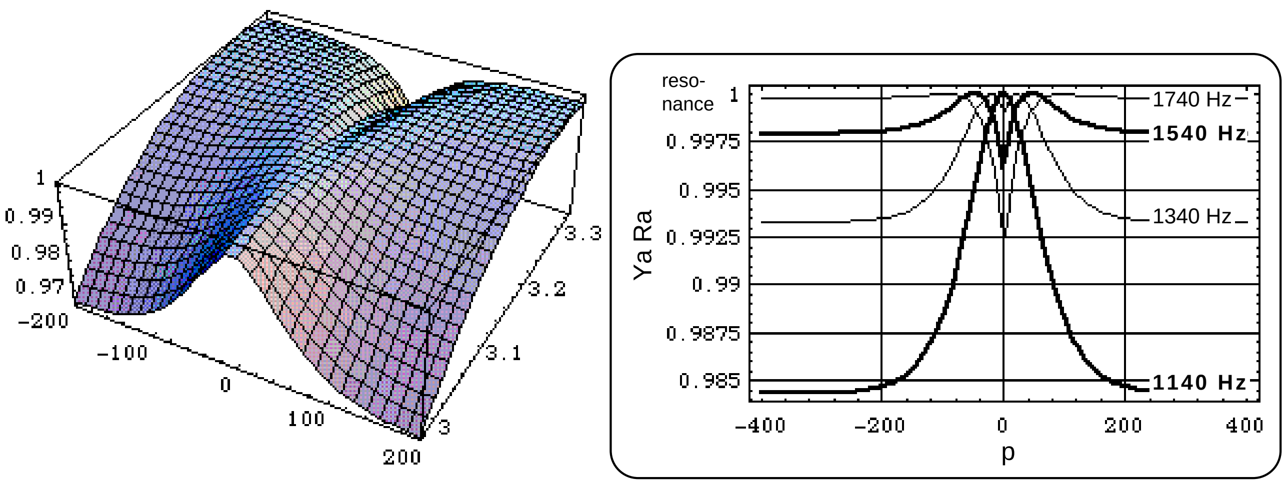


Fig. 13: Detail of Fig. 12 near resonance at zero ear canal pressure p = 0.

Graphical Construction of G_aR_a(log f), B_aR_a(logf)

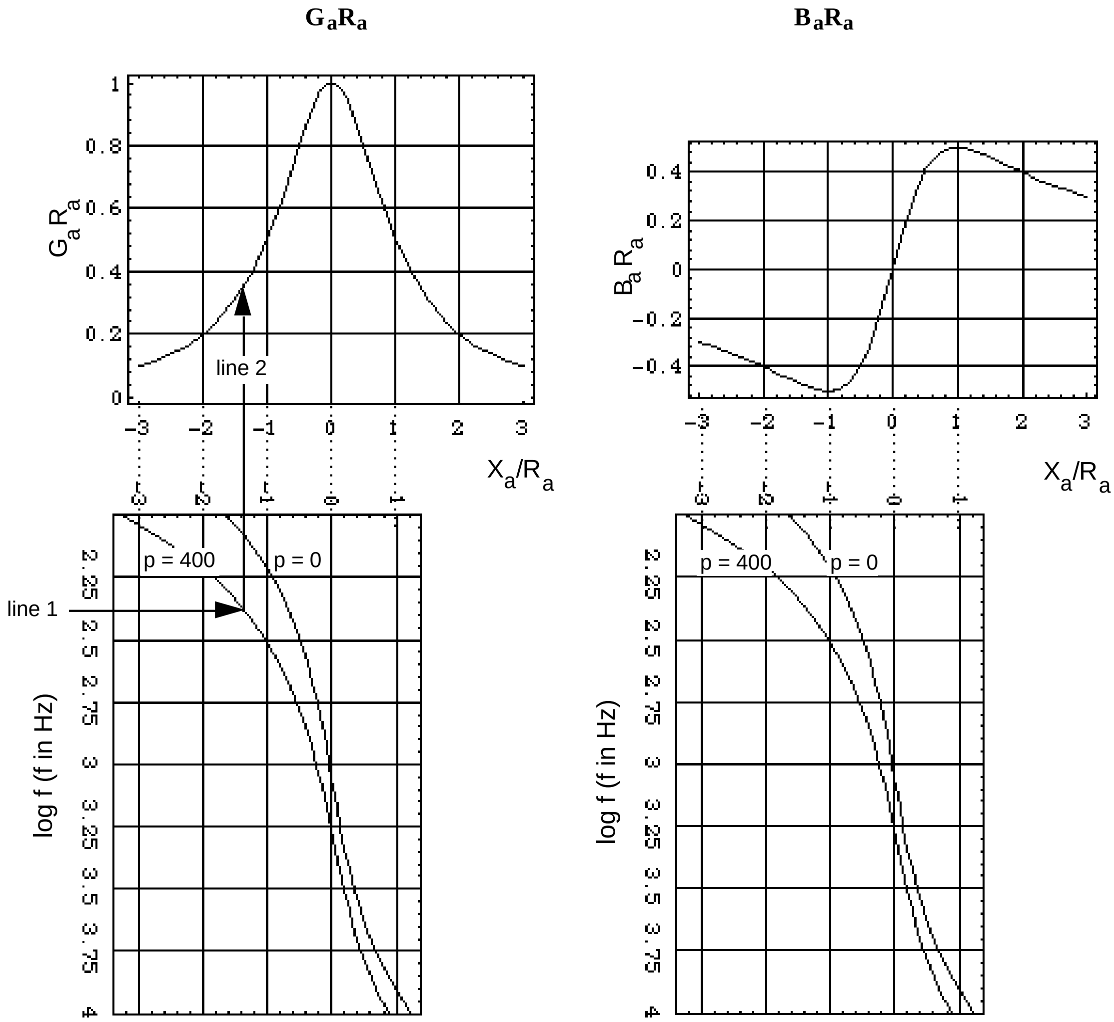


Fig. 14: Graphical explanation of shapes of curves in Figs. 8 - 11:
Lower plots: Xa/Ra as functions of f for fixed p = 0 and p = 400 daPa (see Fig. 6).
Upper plots: G_aR_a and B_aR_a as functions of X_a/R_a (see Fig. 1).
To obtain a value G_aR_a for a given immission frequency f
(1) choose f and read X_a/R_a from lower plot (follow line 1 in direction of arrow),
(2) then read G_aR_a for X_a/R_a (follow line 2 in direction of arrow).

III.2 Coupled Systems
(fit of acoustical behavior with an electrical network model after Zwislocki)

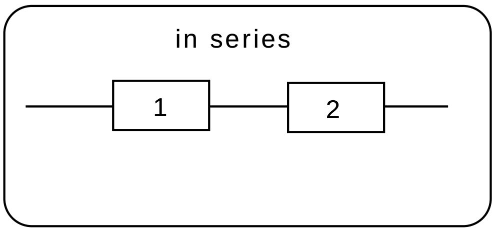


Fig. 15 Electrical system composed of 2 subsystems (1) and (2) arranged in series.

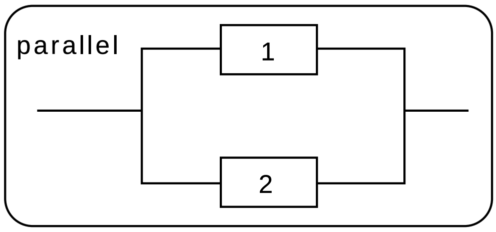


Fig. 15': Electrical system composed of 2 subsystems (1) and (2) arranged in parallel.

Definition 10: complex electrical resistance -"impedance", Z

$$(L_i \frac{d^2}{dt^2} + R_i \frac{d}{dt} + \frac{1}{C_i}) = Z_i \qquad (32)$$

Observation: Ohm's Law for complex resistance.

Let U be the voltage between entrance and exit terminals of a system i and q_i the electric charge in system i. Then the charge q_i is proportional to the applied voltage U:

$$Z_1\, q_1 = U$$
$$Z_2\, q_2 = U \qquad (33)$$

The same is true for a composite circuit:

$$Z\, q = U \qquad (34)$$

Observation:

The flow of charges through subsystems arranged in series is the same in each subsystem:

$$q = \frac{U_1}{Z_1},\ q = \frac{U_2}{Z_2} \qquad (35)$$

Observation:

Charges in parallel subsystems add up in composite circuit:

$$q = q_1 + q_2. \qquad (35')$$

Theorem 6: Composite resistances

The composite resistance Z of a system composed of subsystems arranged in series is:

$$Z = Z_1 + Z_2 \qquad (36)$$

Proof:

Definition of Z: $q = \frac{U}{Z}$ (37)

From (35) follows: $U_1 + U_2 = q\,(Z_1 + Z_2)$. Comparison with (37), the definition of Z (i.e. with $U = q\,Z$) follows $Z = Z_1 + Z_2$.

Theorem 6': Composite resistances

The composite resistance Z of a system composed of subsystems arranged in parallel is calculated as:

$$Y_a = Y_{a1} + Y_{a2} \qquad (36')$$

Proof:

Plugging in observation (35') into Ohm's Law (34)

$Z\,(\frac{U}{Z_1} + \frac{U}{Z_2}) = U$. Simplification yields proof $\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z}$

III.2.1 Example: 2-Component system with subsystems arranged in parallel (Fig. 15')

Data used for calculations (values chosen arbitrarily, i.e. not with respect to a particular electrical middle ear model):
(Mathematica MR 1, 2 "2-Component System")

m_{eff1} := 0.1 g; R_1 := 1000 ohm;
m_{eff2} := 0.01 g; R_2 := 300 ohm;

r_1 := 0.4 cm; r_2 := 0.37 cm;
V_1 := 0.9 cm^3; V_2 := 0.2 cm^3;
ρ_1 := 0.001 g/cm^3; ρ_2 := 0.00129 g/cm^3;
TW = 40 daPa.

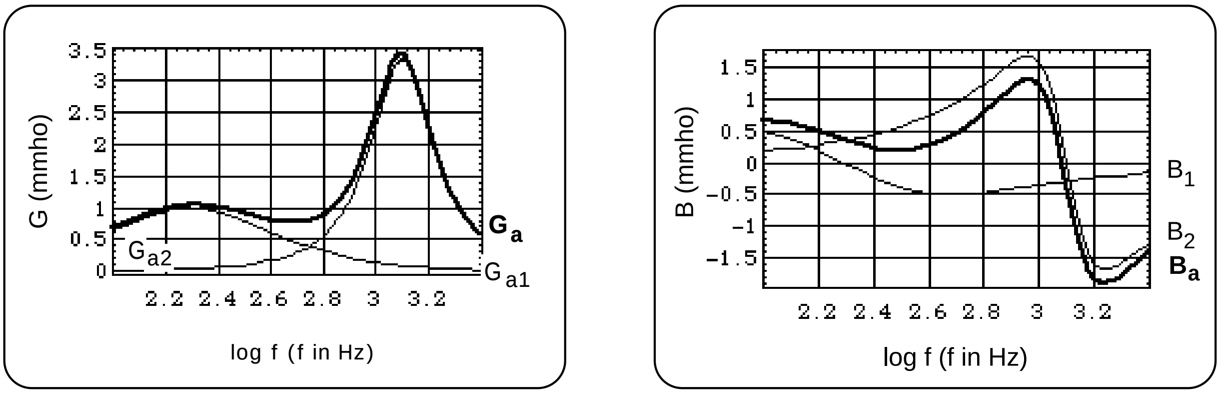


Fig. 16: Conductance G_a and susceptance B_a plotted as functions of immission frequency f. Because of (36') $G_a = G_{a1} + G_{a2}$, and $B_a = B_{a1} + B_{a2}$.

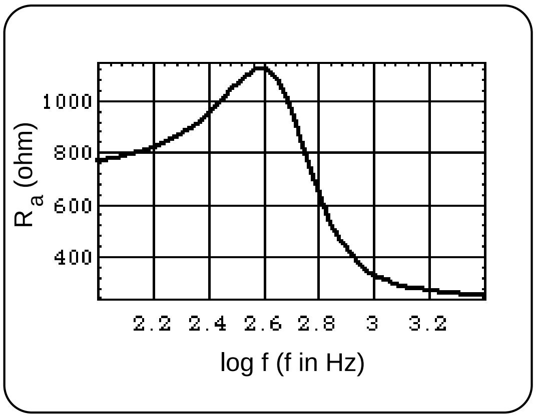


Fig. 17: Resistance of the composite system as a function of immission frequency f.

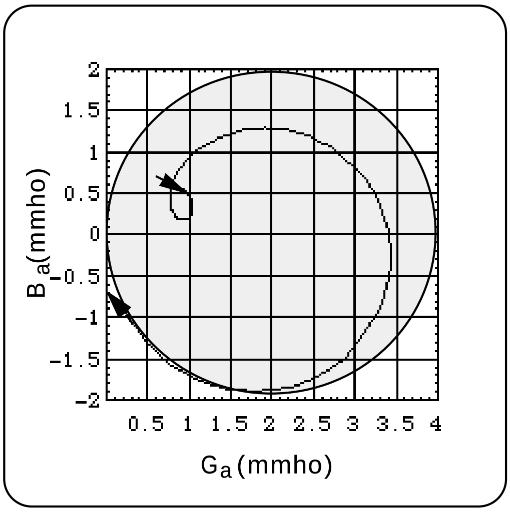


Fig. 18: Oscillation of composite system in $\{G_a, B_a\}$ plane. The point $\{G_a,(f)\ B_a(f)\}$ runs on the curve in the direction indicated by the arrows, when f runs from 100 Hz to 4111 Hz. The circle has been drawn to emphasize non-circular form of curve.

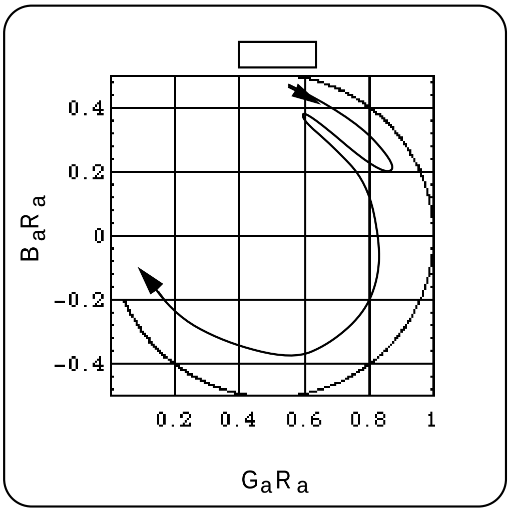


Fig. 19: Oscillation of composite system plotted in {GaRa, BaRa} plane lies on circle with radius 1/2. The reason for this is the linearity of the composite system: The oscillation of each subsystem lies on this circle (see Fig. 2), thus the linear composition of these oscillations lies on that circle, too. The curve drawn by hand indicates how the point $\{G_aR_a, B_aR_a\}$ runs on the circle when f runs from 110 Hz (arrow near {0.6, 0.4}) to 4060 Hz (arrow ending near {0.1, -0.1}).

III.3 Fit of measured tympanometric data with linear model

In Fig. 17-17, R.H. Margolis and L.L. Hunter present a multifrequency tympanogram (R.H. Margolis and L.L. Hunter, Acoustic Immission Measurements, Ch. 17 of *Audiology: Diagnosis,* R.J. Roeser, M. Valente, H. Hosford-Dunn, Thieme, New York, 2000). At an ear canal pressure p = - 250 daPa the tympanic membrane had the highest mobility. G_a and B_a measured at this ear canal pressure are plotted as functions of the immission frequency f in Figs. 20 and 21.

Fig. 22 results when these B_a are plotted vs. G_a.

These data will be analysed with a linear model. This means that the deviation of the curve in Fig. 20 from a circle will be interpreted as resulting from a frequency dependent resistance $R_a(f)$ according to (27). This may or may not be justified. It is simply a method of condensing the measured data into a set of equations (the ones developed in this paper) and corresponding parameters (necessary to evaluate the equations).

After calculating $R_a(f)$ with (27) (Fig. 23), B_aR_a is plotted vs. G_aR_a, resulting in Fig. 24. These data plotted are plotted $\{G_aR_a, B_aR_a\}$ plane. The curve in Fig. 24 drawn by hand indicates how the point $\{G_aR_a, B_aR_a\}$ runs first clockwise and finally counterclockwise on the circle when f runs between 230 Hz (arrow at beginning of clockwise part) and 1930 Hz (arrow at end of counterclockwise part). The circle crosses the abscissa (G_aR_a-axis) at f = 1350 Hz.

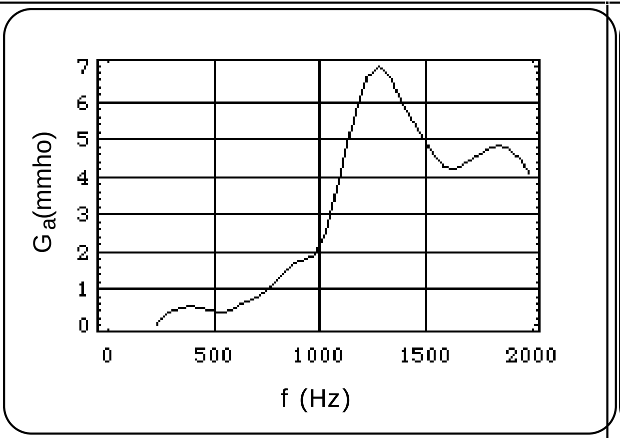


Fig. 20: Conductance G_a as a function of the immission frequency f. The ear canal pressure is - 250 daPa. Data from Margolis and Hunter.

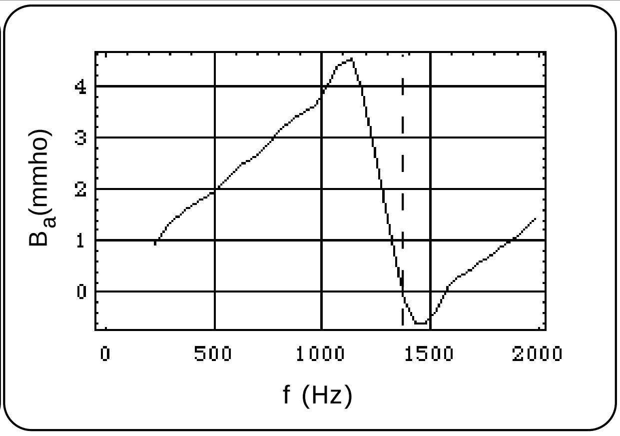


Fig. 21: Susceptance B_a as a function of the immission frequency f. The ear canal pressure is - 250 daPa. Data from Margolis and Hunter. Resonance frequency f_r is defined here as the frequency at which $B_a = 0$ ($f_r = 1350$ Hz, dashed line).

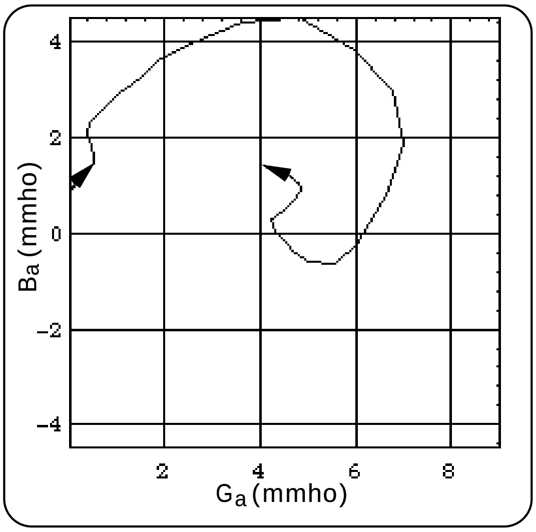


Fig. 22: $B_a(f)$ plotted vs. $G_a(f)$. $B_a(f)$ and $G_a(f)$ as presented in Fig. 20, 21 (from Margolis and Hunter). The curve starts at $f_i = 226$ Hz and ends at $f_i = 2000$ Hz.

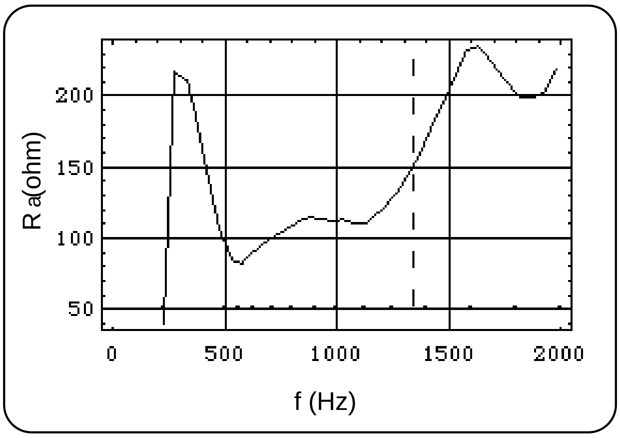


Fig. 23: Resistance extracted from oscillation presented in Fig. 22 with method given by eq. (27). Immission frequencies f_i used by multifrequency tympanometer are marked as dots in lower part of graph. They start at $f_i = 226$ Hz and end at $f_i = 2000$ Hz. Dashed line marks resonance frequency $f_{res} = 1350$ Hz. Sampled frequncies f_i miss resonance f_r.

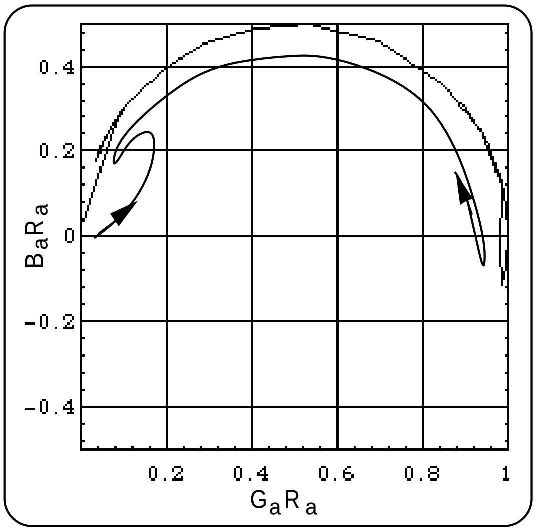


Fig. 24: $B_a(f)R_a(f)$ plotted vs. $G_a(f)R_a(f)$. Data from Margolis and Hunter.